

# Symmetry

To test for symmetry find  $f(-x)$ . Which means substitute  $(-x)$  for all  $x$ 's in the function.

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## **Y-axis symmetry or Even Function**

$$f(x) = 3x^4 - 7x^2 + 6$$

$$f(-x) = 3(-x)^4 - 7(-x)^2 + 6$$

$$f(-x) = 3x^4 - 7x^2 + 6$$

Notice that these 2 functions are equal.

When  $f(-x)$  turns out to be exactly the same as  $f(x)$ , then the function has y-axis symmetry.

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## **Origin symmetry or Odd Function**

$$f(x) = 6x^5 - 8x^3 + x$$

$$f(-x) = 6(-x)^5 - 8(-x)^3 + (-x)$$

$$f(-x) = -6x^5 + 8x^3 - x$$

Notice that ALL 3 of the signs changed.

When  $f(-x)$  turns out to be the exact opposite of  $f(x)$  (all signs change), then the function has origin symmetry.

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## **Neither**

$$f(x) = 6x^5 - 8x^3 + 6$$

$$f(-x) = 6(-x)^5 - 8(-x)^3 + 6$$

$$f(-x) = -6x^5 + 8x^3 + 6$$

Notice that only 2 of the 3 signs changed.

When  $f(-x)$  changes some of the signs from  $f(x)$  but not all of them, then the function has Neither of the symmetries.

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Note: There is another type of symmetry called x-axis symmetry. But this type of symmetry does not occur in functions.

## **X-axis symmetry**

$$x = y^2$$

$$x = (-y)^2$$

$$x = y^2$$

Notice that these 2 relations are equal.

When replacing  $y$  with  $(-y)$  results in the exact same equation that was started with, then the graph has x-axis symmetry.

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